

Engineering Economy

Chapter 6: Comparison and Selection
Among Alternatives

Chapter 6:

Evaluate correctly capital investment **alternatives** when the time value of money is a key influence.

Making decisions means comparing alternatives.

- ❖ In this chapter we examine *feasible design alternatives*.
- ❖ Selecting from among a set of *mutually exclusive* alternatives—when selecting one excludes the choice of any of the others.

Mutually exclusive alternatives (MEAs)

- ❖ We examine these on the basis of economic considerations alone.
- ❖ The alternatives may have different initial investments and their annual revenues and costs may vary.
- ❖ The alternatives must provide comparable “usefulness”: performance, quality, etc.
- ❖ The basic methods from chapter 5 provide the basis for economic comparison of the alternatives.

Apply this rule, based on Principle 2 from Chapter 1.

The alternative that requires the **minimum investment** of capital and produces satisfactory functional results will be chosen *unless* the incremental capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits. This alternative is the *base alternative*.

For alternatives that have a larger investment than the base...

If the extra benefits obtained by investing additional capital are better than those that could be obtained from investment of the same capital elsewhere in the company at the MARR, the investment should be made.

*Please note that there are some cautions when considering more than two alternatives, which will be examined later.)

There are two basic types of alternatives.

Investment Alternatives

Those with initial (or front-end) capital investment that produces positive cash flows from increased revenue, savings through reduced costs, or both.

Cost Alternatives

Those with all negative cash flows, except for a possible positive cash flow from disposal of assets at the end of the project's useful life.

Select the alternative that gives you the most money!

- ❖ For *investment alternatives* the PW of all cash flows must be positive, at the MARR, to be attractive. Select the alternative with the largest PW.
- ❖ For *cost alternatives* the PW of all cash flows will be negative. Select the alternative with the largest (smallest in absolute value) PW.

Investment alternative example

Use a MARR of 10% and useful life of 5 years to select between the investment alternatives below.

	Alternative	
	A	B
Capital investment	-\$100,000	-\$125,000
Annual revenues less expenses	\$34,000	\$41,000

$$PW_A = -100,000 + 34,000(P/A, 10\%, 5) = 28,887$$

$$PW_B = -125,000 + 41,000(P/A, 10\%, 5) = 30,423$$

Both alternatives are attractive, but Alternative B provides a greater present worth, so is better economically.

Cost alternative example

Use a MARR of 12% and useful life of 4 years to select between the cost alternatives below.

	Alternative	
	C	D
Capital investment	-\$80,000	-\$60,000
Annual expenses	-\$25,000	-\$30,000

$$PW_C = -80,000 - 25,000(P/A, 12\%, 4) = -155,933$$

$$PW_D = -60,000 - 30,000(P/A, 12\%, 4) = -151,119$$

Alternative D costs less than Alternative C, it has a greater PW, so is better economically.

Example #1

Your local foundry is adding a new furnace. There are several different styles and types of furnaces, so the foundry must select from among a set of mutually exclusive alternatives. Initial capital investment and annual expenses for each alternative are given in the table below. None have any market value at the end of its useful life. Using a MARR of 15%, which furnace should be chosen?

	Furnace		
	F1	F2	F3
Investment	\$110,000	\$125,000	\$138,000
Useful life	10 years	10 years	10 years
Total annual expenses	\$53,800	\$51,625	\$45,033

Solution

Using a MARR of 15%, the PW is shown for each of the three alternatives in the table below.

	Furnace		
	F1	F2	F3
Investment	\$110,000	\$125,000	\$138,000
Useful life	10 years	10 years	10 years
Total annual expenses	\$53,800	\$51,625	\$45,033
Present Worth @ 15%	-\$380,010	-\$384,094	-\$364,010

The largest value is -\$364,010, indicating that Furnace F3 is the best alternative.

Determining the study period.

- ❖ A *study period* (or *planning horizon*) is the time period over which MEAs are compared, and it must be appropriate for the decision situation.
- ❖ MEAs can have *equal* lives (in which case the study period used is these equal lives), or they can have *unequal* lives, and at least one does not match the study period.
- ❖ The equal life case is straightforward, and was used in the previous two examples.

Unequal lives are handled in one of two ways.

- ❖ Repeatability assumption
 - ❖ The study period is either indefinitely long or equal to a common multiple of the lives of the MEAs.
 - ❖ The economic consequences expected during the MEAs' life spans will also happen in succeeding life spans (replacements).
- ❖ Coterminated assumption: uses a finite and identical study period for all MEAs. Cash flow adjustments may be made to satisfy alternative performance needs over the study period.

Comparing MEAs with equal lives.

- When lives are equal adjustments to cash flows are not required.
- The MEAs can be compared by directly comparing their *equivalent worth* (*PW, FW, or AW*) calculated using the MARR.
- The decision will be the same regardless of the equivalent worth method you use.

For a MARR of 12%, select from among the MEAs below.

	Alternatives			
	A	B	C	D
Capital investment	-\$150,000	-\$85,000	-\$75,000	-\$120,000
Annual revenues	\$28,000	\$16,000	\$15,000	\$22,000
Annual expenses	-\$1,000	-\$550	-\$500	-\$700
Market Value (EOL)	\$20,000	\$10,000	\$6,000	\$11,000
Life (years)	10	10	10	10

Selecting the best alternative.

Present worth analysis → select Alternative A (but C is close).

$$PW_A = -150,000 + 27,000(P/A, 12\%, 10) + 20,000(P/F, 12\%, 10) = 8,995$$

$$PW_B = -85,000 + 15,450(P/A, 12\%, 10) + 10,000(P/F, 12\%, 10) = 5,516$$

$$PW_C = -75,000 + 14,500(P/A, 12\%, 10) + 6,000(P/F, 12\%, 10) = 8,860$$

$$PW_D = -120,000 + 21,300(P/A, 12\%, 10) + 11,000(P/F, 12\%, 10) = 3,891$$

Annual worth analysis—the decision is the same.

$$AW_A = \$1,592$$

$$AW_C = \$1,568$$

$$AW_B = \$976$$

$$AW_D = \$689$$

Example #2

Consider the following two mutually exclusive alternatives for reclaiming a deteriorating inner-city neighborhood (one of them must be chosen). Notice that the IRR for both alternatives is 27.19%.

EOY	Alternative	
	X	Y
0	-\$100,000	-\$100,000
1	\$50,000	0
2	\$51,000	0
3	\$60,000	\$205,760
IRR	27.19%	27.19%

Example #2 continued

- What is the simple payback period for each alternative?
- If MARR is 15% per year, which alternative is better?
- Which alternative would you recommend?
- If MARR is 27.5% per year, which alternative is better?

EOY	Alternative	
	X	Y
0	-\$100,000	-\$100,000
1	\$50,000	0
2	\$51,000	0
3	\$60,000	\$205,760
IRR	27.19%	27.19%

Solution

a) The simple payback period for X is 2 years.
The simple payback period for Y is 3 years.

a) $PW_X (15\%) = \$21,493$
 $PW_Y (15\%) = \$35,291$ *recommend Y

c) Alternative Y

d) $PW_X (27.5\%) = -\$464$ *recommend X
 $PW_Y (27.5\%) = -\$727$

Using rates of return is another way to compare alternatives.

- ❖ The return on investment (rate of return) is a popular measure of investment performance.
- ❖ Selecting the alternative with the largest rate of return can lead to incorrect decisions—do not compare the IRR of one alternative to the IRR of another alternative. The only legitimate comparison is the IRR to the MARR.
- ❖ Remember, the *base alternative* must be attractive (rate of return greater than the MARR), and the *additional* investment in other alternatives must itself make a satisfactory rate of return on that increment.

Ch 6.4: Useful Lives are Equal to the Study Period

Incremental analysis

	Alt. A	Alt. B
Initial cost	-\$25,000	-\$35,000
Net annual income	\$7,500	\$10,200
IRR on total cash flow	15%	14%

Which is preferred using a 5 year study period and MARR=10%?

Ch 6.4: Useful Lives are Equal to the Study Period

Incremental analysis

Base Alternative

	Alt. A	Alt. B
Initial cost	-\$25,000	-\$35,000
Net annual income	\$7,500	\$10,200
IRR on total cash flow	15%	14%

Which is preferred using a 5 year study period and MARR=10%?

Ch 6.4: Useful Lives are Equal to the Study Period

Incremental analysis

Base Alternative

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-\$25,000	-\$35,000	-\$10,000
Net annual income	\$7,500	\$10,700	\$3200
IRR on total cash flow	15%	14%	18%

Which is preferred using a 5 year study period and MARR=10%?

Ch 6.4: Useful Lives are Equal to the Study Period

Incremental analysis

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-\$25,000	-\$35,000	-\$10,000
Net annual income	\$7,500	\$10,700	\$3200
IRR on total cash flow	15%	14%	18%

Which is preferred using a 5 year study period and MARR=10%?

Both alternatives A and B are acceptable—each one has a rate of return that exceeds the MARR. Choosing Alternative A because of its larger IRR would be an incorrect decision. By examining the incremental cash flows we see that the extra amount invested in Alternative B earns a return that exceeds the IRR—so B is preferred to A. Also note...

Ch 6.4: Useful Lives are Equal to the Study Period

Incremental analysis

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-\$25,000	-\$35,000	-\$10,000
Net annual income	\$7,500	\$10,700	\$3,200
IRR on total cash flow	15%	14%	18%

Which is preferred using a 5 year study period and MARR=10%?

Both alternatives A and B are acceptable—each one has a rate of return that exceeds the MARR. Choosing Alternative A because of its larger IRR would be an incorrect decision. By examining the incremental cash flows we see that the extra amount invested in Alternative B earns a return that exceeds the IRR—so B is preferred to A. Also note...

$$PW_A = -25,000 + 7,500(P/A, 10\%, 5) = 3,431$$

$$PW_B = -35,000 + 10,200(P/A, 10\%, 5) = 3,666$$

Why wouldn't we invest in project that gives us higher return?

Incremental analysis

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-\$25,000	-\$35,000	-\$10,000
Net annual income	\$7,500	\$10,700	\$3,200
IRR on total cash flow	15%	14%	18%

Which is preferred using a 5 year study period and MARR=10%?

Why wouldn't we invest in project that gives us higher return?

Incremental analysis

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-\$25,000	-\$35,000	-\$10,000
Net annual income	\$7,500	\$10,200	\$3,200
IRR on total cash flow	15%	14%	18%

Which is preferred using a 5 year study period and MARR=10%?

We assume available capital (\$10,000) will be invested in a project where it will earn annual return = MARR (10%)

Use the incremental investment analysis procedure.

- ❖ Arrange (rank order) the feasible alternatives based on increasing capital investment.
- ❖ Establish a base alternative.
 - ❖ Cost alternatives—the first alternative is the base.
 - ❖ Investment alternatives—the first *acceptable* alternative ($IRR > MARR$) is the base.
- ❖ Iteratively evaluate differences (incremental cash flows) between alternatives until all have been considered.

Evaluating incremental cash flows

- ❖ Work up the order of ranked alternatives smallest to largest.
- ❖ Subtract cash flows of the lower ranked alternative from the higher ranked.
- ❖ Determine if the incremental initial investment in the higher ranked alternative is *attractive* (e.g., $IRR > MARR$, PW, FW, AW all > 0). If it is attractive, it is the “winner.” If not, the lower ranked alternative is the “winner.” The “loser” from this comparison is removed from consideration. Continue until all alternatives have been considered.
- ❖ This works for both *cost* and *investment* alternatives.

Incremental analysis

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-\$25,000	-\$35,000	-\$10,000
Net annual income	\$7,500	\$10,200	\$3,200
IRR on total cash flow	15%	14%	11%

Which is preferred using a 5 year study period and MARR=10%?

Both alternatives A and B are acceptable—each one has a rate of return that exceeds the MARR. Choosing Alternative A because of its larger IRR would be an incorrect decision. By examining the incremental cash flows we see that the extra amount invested in Alternative B earns a return that exceeds the IRR—so B is preferred to A. Also note...

$$PW_A = -25,000 + 7,500(P/A, 10\%, 5) = 3,431$$

$$PW_B = -35,000 + 10,200(P/A, 10\%, 5) = 3,666$$

Example #3

Acme Molding is examining 5 alternatives for a piece of material handling equipment. Each has an expected life of 8 years with no salvage value, and Acme's MARR is 12%. Using an incremental analysis, which material handling alternative should be chosen? The table below includes initial investment, net annual income, and IRR for each alternative.

	Alternative				
	A	B	C	D	E
Capital investment	\$12,000	\$12,500	\$14,400	\$16,250	\$20,000
Net annual income	\$2,500	\$2,520	\$3,050	\$3,620	\$4,400
IRR	12.99%	12.04%	13.48%	14.99%	14.61%

Solution

Alternative A is the base alternative, with an $IRR > MARR$. The next largest investment is in Alternative B, so first examine the incremental investment of B over A. In the table below the IRR of B – A is shown.

	Alternative		
	A	B	B - A
Capital investment	\$12,000	\$12,500	\$500
Net annual income	\$2,500	\$2,520	\$20
IRR	12.99%	12.04%	-20.11%

Alternative B is not better than A — A “wins.”

Solution

The next largest investment is in Alternative C, so examine the incremental investment of C over A. In the table below the IRR of C – A is shown.

	Alternative		
	A	C	C - A
Capital investment	\$12,000	\$14,400	\$2,400
Net annual income	\$2,500	\$3,050	\$550
IRR	12.99%	13.48%	15.86%

15.86% > MARR, so Alternative C “wins.”

Solution

The next largest investment is in Alternative D, so examine the incremental investment of D over C. In the table below the IRR of D – C is shown.

	Alternative		
	C	D	D - C
Capital investment	\$14,400	\$16,250	\$1,850
Net annual income	\$3,050	\$3,620	\$570
IRR	13.48%	14.99%	25.94%

25.94% > MARR, so Alternative D “wins.”

Solution

Finally, examine the incremental investment of E over D.

	Alternative		
	D	E	E - D
Capital investment	\$16,250	\$20,000	\$3,750
Net annual income	\$3,620	\$4,400	\$780
IRR	14.99%	14.61%	12.95%

$12.95\% > \text{MARR}$, so Alternative E “wins,” and we would select Alternative E as the best of these five alternatives.

Comparing MEAs with unequal lives.

- ❖ The repeatability assumption, when applicable, simplified comparison of alternatives.
- ❖ If repeatability cannot be used, an appropriate study period must be selected (the coterminated assumption). This is most often used in engineering practice because product life cycles are becoming shorter.

The useful life of an alternative is less than the study period.

❖ Cost alternatives

- ❖ Contracting or leasing for remaining years may be appropriate
- ❖ Repeat part of the useful life and use an estimated market value to truncate

❖ Investment alternatives

- ❖ Cash flows reinvested at the MARR at the end of the study period
- ❖ Replace with another asset, with possibly different cash flows, after the study period

The useful life of an alternative is greater than the study period.

- ❖ Truncate the alternative at the end of the study period, using an estimated market value.
- ❖ The underlying principle in all such analysis is to compare the MEAs in a decision situation over the same study (analysis) period.

Equivalent worth methods can be used for MEAs with unequal lives.

- ❖ If repeatability can be assumed, the MEAs are most easily compared by finding the annual worth (AW) of each alternative over its own useful life, and recommending the one having the most economical value.
- ❖ For co-termination, use any equivalent worth method using the cash flows available for the study period.

Equivalent Worth Methods

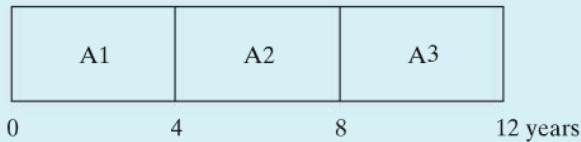
Equate the MEAs annual worths (AW) over *equivalent* lives.

	A	B
Capital Investment	\$3,500	\$5,000
Annual Cash Flow	\$1,255	\$1,480
Useful Live (years)	4	6

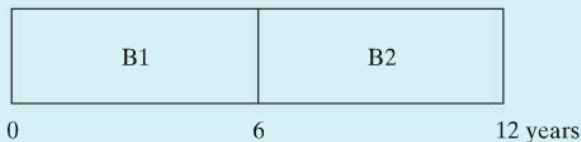
Repeatability & Coterminated Assumptions

Part 1: Repeatability Assumption, **Example 6-7**,
Least Common Multiple of Useful Lives
Is 12 years.

Three cycles of Alternative *A*:



Two cycles of Alternative *B*:



Part 2: Coterminated Assumption, **Example 6-8**,
Six-Year Analysis Period.

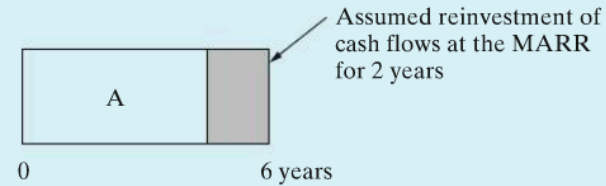


Figure 6-9 Illustration of Repeatability Assumption (**Example 6-7**) and Coterminated Assumption (**Example 6-8**)

We can use incremental rate of return analysis on MEAs with unequal lives.

Equate the MEAs annual worths (AW) over their respective lives. MARR = 10%

	A	B
Capital Investment	\$3,500	\$5,000
Annual Cash Flow	\$1,255	\$1,480
Useful Live (years)	4	6

$$-\$3,500(A/P, i^*, 4) + \$1,255 = -\$5,000(A/P, i^*, 6) + \$1,480$$

Solving, we find $i^*=26\%$, so Alt B is preferred.

Example #4

Consider the following EOY cash flows for two mutually exclusive alternatives (one must be chosen).

MARR=5%

	Lead Acid	Lithium Ion
Capital Investment	\$6,000	\$14,000
Annual Expenses	\$2,500	\$2,400
Useful Life	12 years	18 years
Market Value at end of life	\$0	\$2,800

A. Assuming repeatability, which alternative should be selected?

Solution

Use AW to deal with different useful lives

$$AW_{\text{lead acid}}(5\%) = -\$6,000(A/P, 5\%, 12) - \$2,500 = -\$3,176.80$$

$$AW_{\text{lithium ion}}(5\%) = -\$14,000(A/P, 5\%, 18) + \$2,800(A/F, 5\%, 18) - \$2,400 = -\$3,497.60$$

Select Alternative X (could also calculate PW over 36 years and compare)

Example #5

Consider the following EOY cash flows for two mutually exclusive alternatives (one must be chosen).

	Lead Acid	Lithium Ion
Capital Investment	\$6,000	\$14,000
Annual Expenses	\$2,500	\$2,400
Useful Life	12 years	18 years
Market Value at end of life	\$0	\$2,800

B. Which alternative should be selected if analysis period is 18 years, the repeatability assumption does not apply, and a battery system can be leased for \$8,000 per year after the useful life of either battery is over

Solution

$$\begin{aligned}PW_{\text{lead acid}}(5\%) &= -\$6,000 - \$2,500(P/A, 5\%, 12) - \$8,000(P/A, 5\%, 6)(P/F, 5\%, 12) \\ &= -\$50,767.45\end{aligned}$$

$$PW_{\text{lithium ion}}(5\%) = -\$3,497.60(P/A, 5\%, 18) = -\$40,885.54$$

Select lithium ion battery system (could also calculate AW over 18 years and compare).

Imputed Market Value

For the given study period: Find the PW of the Capital Recovery and the PW of Market Value

	E1	E2
Capital Invest	\$15,000	\$65,000
Ann Exp	\$15,000	\$8,000
Useful Life	5	20
MV	\$8,500	\$14,000
MARR 12%		

PW of CR

1. Find AW of Capital at its own life

$$\$65,000(A/P, 12\%, 20) = \$8,702.12$$

2. Find A of MV at its own life

$$\$14,000(A/F, 12\%, 20) = \$194.30$$

3. Find the Capital Recovery at own life:

$$\$8702.12 - \$194.30 = 8507.82$$

4. Bring into Present Worth AT STUDY PERIOD

$$\$8507.82(P/A, 12\%, 15) = \$57,945.61$$

PW of CR

1. Find AW of Capital at its own life
 $\$65,000(A/P, 12\%, 20) = \$8,702.12$
2. Find A of MV at its own life
 $\$14,000(A/F, 12\%, 20) = \194.30
3. Find the Capital Recovery at own life:
 $\$8702.12 - \$194.30 = 8507.82$
4. Bring into Present Worth AT STUDY PERIOD
 $\$8507.82(P/A, 12\%, 15) = \$57,945.61$

Find PW of MV

5. Find PW of Market Value at the End of study period.
 $\$14,000(P/F, 12\%, 15) = \$2,557.75$

6. MARKET VALUE OF EOY5 =
PW of capital recovery + PW of market value

$$\$57,945.61 + \$2,557.75 = \$60,503.36$$

Bring into A

$$60,503.36(A/P, 12\%, 5) = \$16,507.81$$